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Russell on Negative Judgement

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1 Introduction

Philosophical concern with *negative judgement* has to do with three classes of cases. There are two kinds of *negative existentials*: *general* ('there is no hippopotamus in this room at present') and *singular* ('Loki does not exist'). Additionally, there are *negative predications* ('the table in front of me is not round'). This is the class of negative judgements that traditional logic was mostly concerned with: here 'negative judgement' was defined as a judgement 'in which the predicate is denied of the subject'.¹

These three cases were not clearly delineated until in modern logic. Traditional logic did not acknowledge existential judgements as a separate category; insofar as they were taken into account at all, they were assimilated into the predicative structure. Neither did traditional logic draw a clear distinction between the singular case, on the one hand, and the universal or the particular case, on the other hand, either in predicative or in existential judgements. Furthermore, traditional logic recognized forms of negative predication for which there is no room in modern logic: not only is there the canonical form of negative predication, 'A is not B' (which schoolmen perceived to be equivalent with 'it is not the case that A is B'); there is also term-negation resulting in 'infinite judgements' ('A is not-B'; 'not-A is B').²

Bertrand Russell was an advocate of modern logic. But he did not fully accept the three-fold classification, either. That was because he did not believe that there are any singular existentials. If *a* is a genuine proper name, we cannot meaningfully say either that *a* exists or that *a* does not exist, because such utterances violate 'logical grammar' (the syntax of *Principia*).³ This leaves Russell with just two kinds of negative judgements: negative predications and negative general existentials, including those that are made using definite descriptions.

'Judgement', too, is a problematic notion in the context of mainstream twentieth-century logic. Since Russell was a champion of the new logic, one might expect him to have shunned the notion entirely. And it is true that he had no use for 'judgement' in the sense of traditional logic. In

¹ As in John Newton's textbook from the late 17th century (Newton 1671: Book 1, Chapter XVIII), a characterization repeated *verbatim* by J. S. Mill in his *System of Logic* (1843: Book I, Chapter IV, § 2).

² Cf. Aho and Yrjönsuuri (2009: Sect. 4).

³ See, e.g., Russell (1918a: Lecture 5).

this sense, formal logic is that branch of epistemology which is concerned, roughly, with the formal study of judgements—or their verbal expressions, propositions—and their compounding into inferences.⁴ In modern logic, judgement in this sense was weeded out of logical theory and replaced by a Platonistically understood notion of proposition or sentence, as in Tarski.⁵ Russell's relationship to this development was far from straightforward, however. On the one hand, being a practicing logician, he assumed that propositions *qua* sentences of a formal language are freely available, though he officially regarded propositions as 'incomplete symbols' that are eliminable through contextual definitions. On the other hand, when it comes to the underlying logical ontology, Russell's thinking moved away from Platonism, rather than towards it, and he never accepted an ontology of abstract representational entities after the manner of Bolzano or Frege, which are precursors of 'sentences' in the modern logician's sense. A logician needs propositions primarily to fulfil the role of truth-bearers. But having dispensed with propositions in the 'Russellian' sense that is rather hard to pin down, Russell never returned to the Platonizing attitude that postulates propositions whenever they are needed. Postulations were first excluded by his scientific method in philosophy, which enjoined their replacement by constructions. The abandonment was later strengthened by his naturalism, which construed meaning primarily as a psychological rather logical notion. It is in this broad context that the notion of judgement continues to be potentially important, and it is this sense of judgement that will be the concern of the present paper—Russell himself would later speak about 'belief', a term that sits better with naturalism than 'judgement', even when that latter term is stripped of idealistic connotations.

This paper divides into ten sections. Together, they offer a rough chronology of Russell's conceptions of negative judgement. Section 2 considers some key issues in the theory of negative judgement. Section 3 discusses Russell's account of judgement as a dual relation. Here the focus is on the 'formal' characteristics of judgement, thus understood, and the discussion will compare and contrast Russell's views with Frege's. Sections 4–6 consider Russell's multiple relation theory of judgement. What is of particular interest here is the issue of the ontological ground of negative judgements. For Russell, the question is not just about ontology, but involves logical, semantical and psychological issues as well. This reflects the fact that Russell's thinking about 'judgement' continues to reflect the older philosophical tradition of 'theory of judgement' that draws no sharp disciplinary divides. Of particular interest here is the observation that when such sharp divides begin to emerge in Russell's thinking about judgement, the result is a conception of judgement—

⁴ Russell had even less use for 'judgement' in the broadly idealist sense deriving from Kant; see Russell (1913: 136).

⁵ For this development, see Sundholm (2002).

including negative judgement—that is quite different from mainstream analytic philosophy. In Sections 7–10, I give an outline of this usually neglected development in Russell’s thinking, which involves a new perspective on negative judgement, too.

2 Key questions about negative judgement

Several philosophical questions may be asked about negative judgements, including the following:

- (1) Do negative judgements stand in a *first-order relation* to reality?

The question whether ‘negations’ and ‘affirmations’ are on a par or whether a negative judgement should be seen as a ‘second-order comment on a first-order affirmation’ (Speranza and Horn 2012: 165) is a key issue facing any account of negative judgement. At any rate, it used to be a key issue; most philosophers today would probably accept a positive answer to (1) as a matter of course, but consideration of historical material reveals a rather more varied landscape. Behind the question, there is the following line of thought. On the one hand, judgement has, as we might put it, ‘reference to a reality beyond itself’. On the other hand, we feel that reality is *positive*.⁶ How, then, can a negative judgement even purport to be descriptive of reality? This formulation of the question follows Ledger Wood’s paper ‘The Paradox of the Negative Judgement’.⁷ Here is Russell’s formulation of a similar worry:

We may say that the purpose of knowledge is to describe the world, and that what makes a judgement of perception true (or false) is in general something that would still be a fact if there were no judgements in the world. The yellowness of the buttercup may be taken to be such a fact, and must be mentioned in a complete description of the world. But would there be the buttercup’s not-blueness if there were no judgements? [...] [I]f I see that a buttercup is yellow, I hardly seem to be adding to my knowledge by remarking that it is not blue and not red. *What, then, is meant, in the way of objective fact, by a true negative judgement?* (1948: 137–138; italics added).

⁶ For a recent articulation of this instinct, see Molnar (2000). As in Molnar’s paper, it is common today to articulate the instinct with the help of the notion of *truth-making*. Russell (1918a) is an important precursor of the recent trend.

⁷ See Wood (1933: 412–413).

Russell's prose here suggests that negativity—the buttercup's not-blueness, for instance—is somehow concerned with *judging*; that there would be no negativity in the world if there were no judging minds. Russell in fact came round to thinking that this is the case (see Sect. 9 below). And Wood argues that negative judgements are not about reality in the way that positive judgements are. He claims that the 'paradox of negative judgement'—how can a negative judgement ever be about reality that is exclusively positive—can only be dissolved by reinterpreting negative judgements so as to make them positive, or affirmative:

A negative judgement is equivalent to an affirmative judgement to the effect that some other affirmative judgement is in error. (Wood 1933: 421)⁸

Wood's way out of the paradox is in fact a way of reinterpreting negation in terms of *falsehood*. The general strategy—though not Wood's particular version of it—may receive some support from the fact that modern logic gives propositional negation ('it is not the case that...') a privileged position: Russell, too, was attracted by the reinterpretation, and he was by no means unique in this respect.

Reflection on question (1) leads to further questions regarding the interpretation of negative judgement:

- (2) What is the *structure* of negative judgements?
- (3) How is *truth* explained for negative judgements?

Russell gave different answers at different times to questions (1), (2) and (3), reflecting his changing views on the correct analysis of judgement-facts, i.e., facts of the form '*S* judges that *p*'.

3 The dual relation theory of judgement

Many early analytic philosophers and phenomenologists agreed on the basic schema of judgement:

(DRTJ) Subject + judging + propositional object.

⁸ For a more elaborate statement of this position, see Sigwart (1905: 155).

This is the *Dual Relation Theory of Judgement*: judgement involves a two-place relation of judging between a subject—a judging mind, perhaps—and a propositional object.⁹ The early Russell agreed on the schema, but gave an original interpretation that construes propositional objects as ‘Russellian propositions’, or objective truths and falsehoods, a notion that plays a major role in the *Principles of Mathematics* (1903), for example.

In the context of DRTJ, the problem of how to interpret negative judgement becomes the question of where to put the negation. Negation could be taken to modify the act of judging (negative judgement as *denial*), or else it could be taken to modify what is judged (negative judgement as *negative proposition*). According to traditional logical theory, there are two judgement *qualities*, affirmative and negative: an affirmative judgement is one that ‘asserts something of something’, while a negative judgement is one that ‘denies something of something’.¹⁰ Applied to what is judged, such characterizations are not very useful, because not all propositions are of subject-predicate form, as Russell was wont to emphasise. But the distinction applies to *judgings*. Suppose that propositions are objects of acts. Judging could then be construed as involving a certain kind of predication, namely predication of truth or non-truth (falsehood): when a judgement is made, either a proposition *is accepted as true* or it is *rejected as false*.¹¹

Frege gave an excellent reason for thinking that denial cannot be the only kind of negation: a propositional object—content—that contains a negation need not be asserted at all but may be merely supposed, as in a conditional judgement or in an indirect proof. ‘Negation,’ Frege concludes, ‘attaches therefore to the content, whether or not it occurs in a judgement’ (1879: § 4). Meinong drew a similar distinction between judgements and assumptions, arguing that both involve a ‘definite position within the antithesis of yes and no, of affirmation and negation’, but differ in that judgement further involves the element of conviction while assumption does not (1977: 10–12).¹²

What, then, becomes of denial, once negation is moved inside the propositional object? Let us give the name *quality dualism* to that variant of DRTJ which holds that judgement comes in two

⁹ The propositional object was variously referred to as *Satz-an-sich* (‘sentence-in-itself’; Bolzano), *beurteilbarer Inhalt* (‘judgeable content’; Frege), *Gedanke* (‘thought’; Frege again), *Sachverhalt* (‘state of affairs’; Husserl, Reinach), *Objektiv* (‘Objective’; Meinong), *proposition* (Russell; Moore), and so on. There were important dissenters like Franz Brentano and Anton Marty.

¹⁰ These characterizations go back to Aristotle: see, e.g. *De interpretatione* 17a21–22; *Prior Analytics* 24a15–16.

¹¹ ‘To affirm something simply is to claim that something is true; to deny something is to claim that something is not true’ (Bolzano 1837, Vol. 1, § 23, note 3); cf. Bolzano (1837, Vol. 2, § 142).

¹² Meinong’s distinction is similar to Frege’s, but it is not the same. According to Meinong, both assumption and judgement are *acts* directed towards ‘Objectives’. Frege, on the other hand, has no use for assumptions as acts. All he needs is the notion of an unasserted thought that can occur as a part of a complex thought that is asserted.

kinds: acceptance and rejection; and let us give the name *quality monism* to the view that there is only one judgement quality.¹³ Let us also use some simple symbols to separate these two kinds of judgement and to incorporate into our symbols Frege's insight that there is also negation as a characteristic of content; or as Meinong might have put it, that there is the antithesis of affirmation and negation, quite apart from the distinction between judging and assuming. We then have the following:

- | | | |
|-----|-------------------------------|-------------------------------|
| (1) | - $\frac{3}{4}$ A | acceptance of A (as true) |
| (2) | - $\frac{3}{4}$ A | rejection of A (as false) |
| (3) | - $\frac{3}{4}$ $\emptyset A$ | acceptance of not-A (as true) |
| (4) | - $\frac{3}{4}$ $\emptyset A$ | rejection of not-A (as false) |

According to quality monism, acceptance of a proposition and rejection of its contradictory are *one and the same thing*. On this view, (1) and (4) signify *one and the same act*, and so do (2) and (3); in accepting a proposition, we are *eo ipso* rejecting its contradictory. Frege held this view.¹⁴

This is how he put the point in a manuscript *Kurze Übersicht meiner logischen Lehren* (Frege 1906):

Assertoric force is to be separated from negation, too. For every thought, there is a contradictory one, so that the rejection of one coincides with the acceptance of the other. Judgement is a choice between contradictory thoughts. The acceptance of one and the rejection of the other are one and the same act. There is therefore no need for a special sign for rejection but only for the negation without assertoric force.¹⁵

Frege's point is that since we have to acknowledge negation as a feature of content, and since a logical notation must reproduce everything that is logically relevant, the symbol (3) is to be preferred over (2), and (1) over (4). Thus, introducing a special sign for rejection serves only to conceal content-negation. But if ' $\neg \text{---} A$ ' is just a notational variant of ' $\frac{3}{4} \emptyset A$ ', wherein, then, lies

¹³ The terminology, as well as the symbolism that is used in the text, is adapted from Künne (2010: 572–578).

¹⁴ This view has an important corollary: 'positive judgement' and 'negative judgement' cannot be made sense of in isolation; what does make sense is the twin-notion of a proposition and its contradictory; see here Künne (2010: 563).

¹⁵ Frege (1906: 214; my translation).

the content-negation in the former symbol? Content-negation can be seen to be present in ‘ $\neg \frac{3}{4} A$ ’, too, when it is realized that what the downwards arrow symbolizes is, roughly, the complex predicate ‘rejection of... as false’ or ‘rejection of ... as not-true’. The notion of rejection, when analysed in accordance with DRTJ, thus combines features both of act and of content, and these ought to be kept separate not only in logical theory but also notationally. Hence, ‘ $\neg \frac{3}{4} A$ ’ is more properly written as ‘ $\neg \frac{3}{4} \emptyset A$ ’, which is what Frege argues for in the above passage.

What Frege has to say in the passage gives a precise formulation of quality monism, not an *argument* for that view, or against quality dualism. If there are compelling logical reasons for introducing assertion and rejection as distinct activities, then they should be distinguished. Frege found no such reasons and he stuck to quality monism.¹⁶

Russell considered the issue of quality dualism and monism in his 1904 review of Meinong. There he accepts quality dualism in a limited sense. He argues that ‘[t]o deny a proposition is not the same as to affirm its denial’ (1904: 447). That is, (2) is not the same as (3): rejection of a proposition must not be identified with acceptance of its negation. This is quality dualism. But he also claims that the distinction is logically irrelevant and has psychological relevance only: ‘[P]sychologically, it would seem, there are two states of mind which both have p for their object, one affirming and the other denying; and two other states of mind, having not- p for their object, one affirming and the other denying’ (1904: 447). These are just the four cases that we distinguished above.

Russell (1904) gives three arguments to support quality dualism. The first is the most interesting, and it concerns the nature of assumption in Meinong’s sense:

Given any proposition p , there is an associated proposition not- p . Either of these may, as Meinong points out, be merely supposed or assumed. But when we deny p , we are not concerned with a mere assumption, and there is nothing to be done with p that is logically equivalent to assuming not- p . (1904: 447)

The passage contains two reasons, or possibly one reason with two connected parts, against quality monism. Since I have been unable to attach any clear meaning to the second part—‘there is nothing to be done with p that is logically equivalent with assuming not- p ’—I shall discuss only the first part. Russell does not elaborate what he has in mind here, but plausibly he is just pointing out that

¹⁶ See Smiley (1996). Frege’s reasons for quality monism and against quality dualism are found in Frege (1919).

there is a clear sense in which a denial is indeed concerned with a judgement, and not with a mere assumption. This happens when a judgement involves, as Adolf Reinach put it, a ‘polemical taking up of a position against another judgement’.¹⁷ If this is correct, it gives a weighty reason for quality dualism.

The point looks perfectly valid, as far as it goes. Reflection on Frege’s version of quality monism shows, however, that the polemical sense of denial is not the only legitimate one. Frege’s conception of judgement does not recognize negative judgement in this ‘polemical’ sense. Instead, he aligns judgement with the *acquisition of knowledge* and conceptualises it as the end point of a process of (ideal) *justification*.¹⁸ This puts Frege’s notion of judgement in a quite specific dialectical context. Judgement is now to be conceived of as an answer to a propositional question – say, ‘Is 649 prime?’ – which demands, as Frege (1919: 362) puts it, that we should either acknowledge as true the thought that 649 is prime or reject it as false. This acknowledgement in turn takes the form of a justification (ideally, a proof) of whichever of the two alternatives is the correct one; and it terminates in a judgement signalling the recognition of one content as true, which is simultaneously a recognition of its contradictory as false.

Given this, it is easy to see why Frege, rather than construing negative judgement polemically, should see it as involving a mere assumption in his sense and why it was so natural for him to accept quality monism and take ‘- $\frac{3}{4}$ $\emptyset A$ ’ as expressing the proper logical form of rejection. The polemical context, on the other hand, is simply one where one judgement (‘- $\frac{3}{4}$ 649 is prime’) is met by a ‘polemical rejoinder’ (‘ $\frac{3}{4}$ 649 is prime’). Here, it is inevitable to introduce acceptance and rejection as distinct acts. *Contra* Russell, it must be emphasised, of course, that the difference between knowledge-acquisition and polemical judging does not belong to psychology.

Russell’s second argument against quality monism is an argument from ‘direct inspection’; this inclines him to think that ‘the state of mind in which we reject a proposition is not the same as that in which we accept its negation’ (1904: 447).¹⁹ His third argument concerns the status of the

¹⁷ Reinach (1911: 357). Reinach’s example of the polemical taking up of a position is the judgment that the king was not energetic, as uttered by a historian who is expressing his opposition to some other historians’ assertion that the king was energetic. The polemical sense contrasts with a ‘mere positing’, which might occur, for example, in the course of a ‘simple portrayal’ of a historical figure (*ibid.*).

¹⁸ This is ultimately a reflection of an epistemic conception of logic. When we judge in Frege’s sense, we are acknowledging the truth of something on the basis of something else that has already been recognized as true; that is, we are *inferring*. And logic is concerned with the discovery of the laws for this kind of justification; cf. Frege (1879–91: 3).

¹⁹ In the *Theory of Knowledge* manuscript of 1913, Russell gives the argument from introspection as the weightiest reason to think, ‘though with no great certainty’, that *disbelief* is an unanalysable

law of the excluded middle. The law may be stated in the provisional form, ‘If p is denied, not- p must be asserted’. This formulation, Russell admits, is not ultimate, but the point is just that the law is a significant proposition and not a mere tautology, which it would be, if accepting a proposition and rejecting its contradictory were literally one and the same act (*ibid.*). This argument seems to depend upon the classical, or pre-Wittgensteinian, conception that a tautological judgement amounts to ‘saying the same thing twice over’ and is therefore not quite a judgement at all.²⁰ If asserting a proposition and denying its contradictory were literally one and the same act, then the law of the excluded middle would be an instance of ‘if A , then A ’; but—and here I am adding to Russell’s argument—since we see that this is not the case, the claim about strict identity cannot be upheld.

These arguments and hence Russell’s quality dualism pertain only to what he sees as the psychology of judgement. In his logical practice, Russell endorsed quality monism in its Fregean form, and for much the same reason. Logic in the fundamental sense is concerned with the principles of correct inference, and this requires the recognition and labelling of the different positions that propositions may have in inference. This is signalled by Russell’s use of the ‘assertion sign’, ‘ \vdash ’. It is derived from Frege by coalescing Frege’s judgement stroke and content stroke (Frege 1879: § 2) into a single symbol. The sign is followed by an appropriate number of dots indicating the range of the assertion, and thus it signals the difference between an ‘asserted’ proposition and one that is merely considered. The intuitive content of the assertion sign remains somewhat unclear, but as negation always goes with what follows the sign, this is clearly a version of quality monism. As far as logical theory is concerned, then, ‘negative judgement’ amounts roughly to the *assertion of a negative proposition*.²¹

One striking feature of Russell’s treatment of negative judgement in the context of DRTJ is that he identifies negation with falsehood or non-truth. In Russell (1906a: *1.5), negation is explained as follows:

relation, rather than belief in the contradictory (1913: 142). ‘Disbelief’ will be mentioned again below, in Sect. 5 and at the end of Sect. 8.

²⁰ For a discussion of ‘tautology’ and its history, see Dreben and Floyd (1991).

²¹ Russell (1906: *1.1) suggests that ‘ \vdash ’ can be read as meaning ‘it is true that’, but he adds that ‘philosophically this is not what it means’ (*ibid.*); the characterization is repeated in *Principia* (Whitehead and Russell 1910: *1). This may be what inspired Wittgenstein to his misleading criticism of ‘assertion’ as signalling merely that ‘the authors [i.e., Frege and Russell] hold as true the propositions marked this way’ (1922: 4.442). For Frege and Russell, assertion belongs to logic, and is not a marker of an attitude.

Negation. The proposition “ p is not true” is expressed by $\sim p$. Thus “ $\sim p$ ” will be true if p is not a proposition, and if p is a false proposition; it will only be false if p is a true proposition.

A feature that stands out about this explanation is that Russell wants to have propositional functions defined for all entities: for ‘ $\sim p$ ’ to be well-defined, it has to yield determinate values even for ps that are not propositions.²² This also explains why Russell uses ‘not true’, rather than ‘false’; ‘ $\sim p$ ’ will be true for non-propositional ps , because (roughly) it can be truly said of entities like Socrates and the Piccadilly that they are not true without implying that they are false. The explanation has the appropriate consequence that the negation of a true proposition is false and the negation of a false proposition is true (a proposition that is false is *a fortiori* not true). But although ‘true’, ‘false’ and ‘not true’ figure in it, the explanation is not a semantic explanation of negation in terms of truth-conditions, which would be the standard way of forging a connection between negation and falsehood. Rather, the explanation identifies negation with non-truth, and does so in a way that renders the latter notion explanatorily primary.

This is a somewhat surprising move, as it seems to turn the natural order of explanation upside down. However, apart from technical considerations, which must be ignored here, there is a philosophical rationale behind the procedure, which is to be found in Russell’s non-standard construal of DRTJ. Russell was always sensitive to the correspondence intuition, according to which a judgement is true or false because something outside the judgement makes it so. But as long as he accepted DRTJ, he resisted the intuition and attributed it to an erroneous notion of judgement as a complex idea that resides in the mind and corresponds, if true, with facts.²³ The true view, he argued, was that a true judgement is simply that a judgment consists in a subject standing in the relation of judging to an object. Recall that the object is a Russellian proposition, which means that if a judgement is true, it has quite literally an objective truth as one of its parts, and there is therefore nothing outside the judgement to ‘make it true’; there is no entity or fact separate from the true judgement that could be referred to as the ground of truth. On this view, truth—and ‘by parity of form’, falsehood, too—must be accepted as primitive features of propositions. And if falsehood is primitive, it might as well be put to a good use in an explanation of negation.

²² This requirement is a consequence of Russell’s ‘universalist’ conception of logic, one that he shares with Frege (cf. Frege 1893: § 6). *Principia*’s treatment of negation is different on this point; cf. Whitehead and Russell (1910: 93).

²³ For Russell’s criticism of truth as correspondence, see Russell (1905: 492–494).

4 Russell's reasons for rejecting the dual relation theory of judgement

Russell eventually rejected DRTJ for logical as well as philosophical reasons. The latter may be condensed into the following line of thought:²⁴

- (1) Judgement is either true or false; a theory of judgement must therefore allow for the duality of truth and falsehood.
- (2) Suppose that a judgement is a two-place relation between the judging mind and a single entity, which is complex.
- (3) Suppose that the entity introduced in (2) is the sort that is expressed by '*a*'s being *F*'.
- (4) But this rules out falsehood; if there *is* such an entity as *a*'s being *F*, then *a is F*, and the judgement cannot be false.²⁵
- (5) Suppose, on the other hand, that the entity introduced in (2) is the sort that is expressed by 'that *a is F*'. *Prima facie*, this kind of entity allows for the duality of truth and falsehood.
- (6) But such entities are mysterious: they are unities (complexes with constituents) and are either true or false; but neither characteristic allows for elucidation or explanation.
- (7) Therefore, judgement cannot be a two-place relation between the judging mind and a single entity.

Consider Othello's judgement that Iago is faithful. The above line of thought may be continued by noting that what there unquestionably *is*, besides Othello, are fidelity and Iago. And Othello can stand in a relation to them by, say, *ascribing the former to the latter*. 'Ascription' here is a placeholder for any 'propositional cognitive relation' (Russell 1913: 110), like judgement, whose logical form is thus brought out more clearly by construing it as an instance of this abstract notion. There is thus the following contrast:

DRTJ Othello believes [that Iago is faithful];

MRTJ Othello ascribes fidelity to Iago.

In the latter case, which is the *multiple relation theory of judgement* (MRTJ), judging is construed as a three-place relation between Othello (his judging mind, more properly), fidelity (a universal)

²⁴ For Russell's own discussion of the matter, see Russell (1910: 150–153; 1912: Chapter 12). In assessing the line of thought, it must be kept in mind that representational entities were not yet taken seriously by Russell.

²⁵ If, for example, there is such an entity as Desdemona's love for Cassio, then, surely, she does love him.

and Iago. The adicity of the ascription-relation varies, depending on what is ascribed to what. This lead Russell to say that the relation is multiple, by which he meant that its adicity is variable but greater than 2.²⁶ Its terms, apart from the subject, are the entities that DRTJ treated as constituents of propositions. A more useful description, though, is that, according to MRTJ, they are the *constituents of the fact or complex* which, if it is there, makes the ascription correct (true). Thus, MRTJ professes to dispense with the two mysteries found at the heart of DRTJ: there are no propositions; and although there are ‘facts’ or ‘complexes’, there is nothing mysterious about their unity, because there is only one kind of unity, namely that which constitutes truth.²⁷

5 Judgement as a multiple relation

In dispensing with propositions, MRTJ attributes truth and falsehood to concrete instances of the abstract ascription-relation. Accordingly, judgement as a truth-bearer is now construed as a *judging*; that is, judgement is an act, or better, the *product* of an act, and hence a particular dated *event* (Russell 1913: 136).²⁸ Now, though, Russell prefers the terminology of ‘belief’, because that term lacks the idealistic connotations of ‘judgement’.

Substituting judgement in the sense of judging for the act/object or act/content distinction raises a number of questions about how the work that was done by the distinction is done within the new framework.²⁹ Negative judgement offers a case in point. The different existing versions of MRTJ are almost exclusively concerned with the simplest case of ‘atomic propositional thought’ or elementary judgement; molecular thought, including negative thought, is only very lightly touched upon by Russell.³⁰ Nor is it clear how the simple recipe given above—Othello ascribing fidelity to

²⁶ We might say that, according to MRTJ, ascription is a multigrade relation, that is, that it is of variable adicity. Russell, however, was inclined to say that because of this variability, there is not one but many ascription relations; that belief, for example, is ‘systematically ambiguous’ (Russell 1918a: 199). Such a view leads to well-known difficulties. For instance, we can no longer speak about (absolutely) *everything* that Othello believes, because there is no such totality to speak about.

²⁷ This is not to say that there is no problem of unity for Russell. But the new theory of judgement professes to make the problem tractable, because it recognizes only one kind of unity, that which constitutes fact, whereas on Russell’s version of DRTJ, there are both fact unities (true propositions) and propositional unities (propositions irrespective of their truth-values).

²⁸ Or ‘complex’ (Whitehead and Russell 1910: 43) or ‘fact’ (Russell 1918a); the terminology varies, depending on how such entities are classified.

²⁹ Cf. van der Schaar (2015).

³⁰ MRTJ is mentioned briefly in Russell (1907), and is definitively advocated in Russell (1910), in the Introduction to *Principia* and in Russell (1912: Chapter 12). It was meant to constitute the conceptual core of the *Theory of Knowledge* project in which Russell was engaged in 1913, but that project came to an abrupt end, apparently because of Wittgenstein’s criticisms. MRTJ’s subsequent history is a somewhat controversial issue, but what is clear is that it was definitively rejected in 1919, when Russell adopted a psychological account of belief (Russell 1919a). The complex

Iago—is to be applied to negative judgement. What are the entities to which Othello is related if he believes that Iago is *not* faithful? What does Othello ‘ascribe’ to what in this case? Or is negative judgement rather a matter of a special kind of ascription-relation holding between Othello, Iago and fidelity?

The first option tallies well with the conclusion that Russell drew in 1913, namely that even in elementary judgements, propositional thought must involve acquaintance with a specific *logical form* (Russell 1913: Part II, Chap. I). On this view, when Othello judges that Iago is faithful, he is related not only to Iago and fidelity but to the ‘form of dual complexes’ (1913: 114):

Judges (*o*, fidelity, *i*, **f** *x*).

What would be more natural, then, than to introduce different *molecular logical forms*—negative, disjunctive, general, as need be—to cover the relevant non-atomic cases? This would give us something like:

Judges (*o*, jealousy, *i*, **¬f** *x*).

A problem with this is that Russell’s attempts to make sense of logical forms as entities that are (i) objects of acquaintance but also (ii) ways in which entities could be combined were not particularly enlightening.³¹ Moreover, we do not know what he may have thought about the suggested extension of the original recipe.³² The *Theory of Knowledge* manuscript breaks off at the crucial point where Russell was about to move from atomic to molecular thought and where he would have taken up the question.

The other suggestion is mentioned by Griffin (1985: 218). He notes Russell’s brief discussion of *disbelief* in the 1913 manuscript (Russell 1913: 142–143), and argues that Russell sought to treat negative judgement by means of a new multiple relation, *denial* or *disbelief*:

Disbelief (*o*, jealousy, *i*, **f** *x*).

developmental history of MRTJ is not yet very well understood in spite of considerable scholarly efforts: Griffin (1985) is a classic in this literature, while Carey (2007), MacBride (2013) and Lebens (2017) are examples of the more recent contributions. Landini and Elkind (2018) offers fresh insights.

³¹ Russell (1913: Part II, Chapter I).

³² Wrinch (1919) works out the extension in some detail, though not so much to prove that the multiple relation theory is correct, as to ‘investigate how far this theory of judgment could be made satisfactory’ (*ibid.* 319).

It is not clear, however, that Russell had in mind this role for disbelief. One difficulty with interpreting negation as disbelief is an instance of the Frege-point: negation may occur unasserted, and in such cases it cannot be interpreted as disbelief—as Russell (1906b: 321) points out in a different context. Furthermore, Russell came to think that *understanding* is the most basic propositional cognitive relation, presupposed in all others.³³ Since Othello can understand what it would be for Iago not to be faithful, it would seem that negation cannot reside in the act of disbelief alone.

The idea that judging involves a ‘polarity’ of attitudes, though it may look like a non-starter, does reappear in Russell’s psychological theory of judgement (see Sect. 8 below). A more prominent topic in MRTJ, however, is the question of how *truth* is explained for negative judgement.

6 Judgement and truth as correspondence

MRTJ is self-consciously a version of the correspondence theory of truth. Having given up DRTJ, Russell was now in a position to pay heed to the correspondence intuition that we have about the concept of truth:

We feel that when we judge truly some entity “corresponding” in some way to our judgment is to be found outside our judgment, while when we judge falsely there is no such “corresponding” entity (Russell 1910: 152).

The intuition aligns truth with existence and falsehood with non-existence. Although dubbed a ‘feeling’, it is more than a general intuition about truth (‘correspondence with reality is what truth consists in’). First, the entities referred to are readily construed as *facts*. If not, it is difficult to see how the simple alignment could be held on to; as the case of negative judgement shows, sometimes truth goes with non-existence (‘there are no unicorns’) and falsehood with existence (‘there are no dogs’). The feeling thus gives way to a more informative formulation:

[A] “fact” may be described as what there is when a judgment is true, but not when it is false (Russell 1913: 79).

Second, Russell’s intention is to define the corresponding fact, a complex entity, as a function of the belief:

³³ Russell (1913: 110).

[W]e may define *truth*, where [elementary] judgments are concerned, as consisting in the fact that there is a complex *corresponding* to the discursive thought which is the judgment. That is, when we judge “*a* has the relation *R* to *b*,” our judgment is said to be *true* when there is a complex “*a*-in-the-relation-*R*-to-*b*,” and is said to be *false* when this is not the case. (Whitehead and Russell 1910: 43; italics in the original)³⁴

Thus, there are the following conditions of truth and falsehood:

- (C-true) a judgement is true « there is a fact such that the judgement corresponds to it;
- (C-false) a judgement is false « there is no fact such that the judgement corresponds to it.

A much later formulation of these conditions is found in *Human Knowledge*, showing that Russell continued to accept their substance even towards the end of his philosophical career:

I come now to the definition of “truth” and “falsehood”. Certain things are evident. Truth is a property of beliefs, and derivatively of sentences which express beliefs. Truth consists in a certain relation between a belief and one or more facts other than the belief. When this relation is absent, the belief is false. (1948: 164–165)³⁵

If one thinks that truth consists in correspondence, it is natural to unpack this in terms of (C-true) and (C-false); I will therefore refer to the two conditions together as the ‘standard account’. Russell clearly accepted the standard account in *Principia* and *Human Knowledge* (and elsewhere, too). But the full story is rather more involved. To begin with, there are several questions that need to be asked about (C-true) and (C-false). One question is, what is their *scope*? Do they apply without restrictions to judgements of every kind and type? Russell is well known for being a ‘logical atomist’, and for a logical atomist the question has quite special importance. Another question concerns *negative judgements* and their relation to (C-true) and (C-false). These questions are closely related, because negative judgements are an important potential rationale for limiting the

³⁴ This is formulated in terms of ‘complex’, rather than ‘fact’; around 1910–13, Russell used the terms more or less interchangeably. Thus, when Russell explains ‘fact’ in 1913, the context of that description is this: ‘A “complex” is anything analysable, anything which has *constituents*. When, for example, two things are related in any way, there seems to be a “whole” consisting of the two things so related [...]. It may be questioned whether a complex is or is not the same as a “fact”, where a “fact may be described as what there is when a judgment is true, but not when it is false. [...] However this may be, there is certainly a one-one correspondence of complexes and facts, and for our present purposes I shall assume that they are identical’ (Russell 1913: 79–80). For further discussion, see Sect. 9 below.

³⁵ By the last sentence, Russell clearly intends to capture (C-false); on the correspondence conception, the relation between a belief and the relevant fact is an *internal* relation and can therefore be absent only if either or both of the terms are absent.

scope of the standard account. Yet another question has to do with how substantial the relevant notion of ‘fact’ is. The standard account quantifies over facts and construes correspondence as a genuine relation—even if it is also an internal one. These features presuppose that the terms of the correspondence relation are existent entities: Russell’s talk of ‘complex’ is indicative of this. Other conceptions of fact are available, however, and they are relevant for the development of Russell’s actual views, too (the matter is discussed in Sect. 9 below).

In linking truth with existence, the standard account faces a version of Wood’s paradox of the negative judgement. Like Wood, Russell, too, felt that reality is in some sense *positive*:

The universe consists of objects having various qualities and standing in various relations.
(Whitehead and Russell 1910: 43)

But this leads to a difficulty, when it is combined with the standard account. ‘There are no griffins’ is true, but if the universe consists of objects having qualities and standing in relations, it is not evident how that negative judgement *can* be true. Considered on its own, the standard account points in a different direction: ‘there are no griffins’ is true; therefore, there is a fact which makes the judgement true, but this is the *negative fact* that there are no griffins.

Russell is well-known for his endorsement of negative facts. The reason why he accepted negative facts, however, is not to be found in the standard account—he never accepted an unrestricted use of the standard account. And the inference from (C-true) to negative facts is easily blocked by a simple restriction of the scope of the standard account. This is the *semantic explanation of negation*: ‘*not-p*’, if true, is not made true by a negative fact, but is true because its contradictory, ‘*p*’, is false, as can be seen by consulting the standard truth-table explanation of negation. Arguably, this is a special case of what holds quite generally, namely, that the truth-value of any logically complex proposition is explained in terms of the truth-values of its elementary (‘atomic’) part or parts. The lesson is that (C-true) and (C-false) should be taken to apply to the elementary case only. One is not necessarily committed to negative facts just because one accepts the standard account.

We are accustomed to thinking of this as *the* logical atomist conception of truth. It is best known from the *Tractatus*. But if we accept the semantic explanation and explain that ‘*not-p*’ is true because ‘*p*’ is false, we face the question: *What, then, makes a proposition false?* (C-false) helps us here, of course, but when the logical atomist Russell addressed the question, he gave quite a different answer:

I think you will find that it is better to take negative facts as ultimate. Otherwise you will find it so difficult to say what it is that corresponds to a proposition. When, e.g., you have a false positive proposition, say “Socrates is alive”, it is false because of a fact in the real world. A thing cannot be false except because of a fact, so that you find it extremely difficult to say what exactly happens when you make a positive assertion that is false, unless you are going to admit negative facts. (Russell 1918a: 189–190)

Russell is here actually arguing against Raphael Demos’ account of negative judgement in terms of incompatibility.³⁶ For our purposes, however, there is a more interesting contrast with the standard account of truth as correspondence. Russell, it was argued above, held that the explanation of why a belief is true or false involves the specification of a determinate something. Looking at (C-false), we see that it refers to an *absence of a fact* as the condition of a belief’s falsity. What is supposed to be absent is a specific fact, a truth-maker for a judgement, and the task is to specify the absent fact as a function of the belief, a task that consists in finding a description of the fact which, if there is one, makes the belief true.³⁷ One may suspect that a *problem of reference* is lurking here; Russell thought so and took the problem very seriously.

A certain kind of solution to the specification problem is available. It was adopted by Moore in his *Some Main Problems of Philosophy*:

For any belief p (p is true « there is a unique x : fact (x) and p refers to x and x exist)

For any belief p (p is false « there is a unique x : fact (x) and p refers to x and x does not exist).³⁸

Here the fact is specified as a function of a belief, because the definitions of truth and falsehood make use of an unanalysed relation of *reference*: a belief refers to a unique fact, and every different belief refers to a different fact; one might find Moore’s use of ‘fact’ odd, but he is just using the

³⁶ That is, it is better to take negative facts as ultimate than to accept Demos’ analysis of negation. According to Demos, ‘not p ’ is a ‘relational modification of p ’ and means ‘opposite of’ or ‘contrary of’ or ‘inconsistent with’ (1917: 190). On a natural reading of what Demos was after, the point is, for example, that ‘this is not red’ is true because there is some positive true proposition (say ‘this is white’) that is contrary of or incompatible with ‘this is red’, thus entailing its negation. Russell criticized Demos on the grounds that ‘opposite of’ or ‘contrary of’ or ‘inconsistent with’ can none of them hold without negative facts: ‘this is white’ is incompatible with ‘this is red’ and therefore the former can be used to deny the latter; but, Russell contends, this is just because what is white is also *not* red, and this has to be a fact; see Russell (1918a: 187–188; 1919a: 280–281).

³⁷ Cf. here Russell (1948: 165–166).

³⁸ See Moore (1953: Chapters 14 and 15).

word in the sense in which facts divide into those that exist and those that do not exist. He professes to be unable to say exactly what a belief and having a belief are in the sense of analysing these notions. But he argues that we nevertheless do know, for any number of *ps*, what it is to have the belief that *p*; and to the extent that we know this, we also know what fact it is whose existence or non-existence makes the belief true or false, respectively.³⁹ Moore's theory is a simple version of the standard account. He applies the theory to all kinds of belief, and he does this quite consciously, with the consequence that since there are negative truths, conditional truths, and so on, there are also negative facts, conditional facts, and so on. Moore is not overly troubled by this (Moore 1953: 268), but he is troubled by the consequence, which he also sees quite clearly, that the theory commits him to entities which do not exist (*ibid.* 263).⁴⁰

The *Tractatus* account of truth looks like a structural variant of Moore, with the difference that it is restricted to 'atomic propositions':

For any atomic proposition, *p* (*p* is true « there is a unique *x*: Sachverhalt (*x*) and *p* pictures *x* and *x* obtains);

For any atomic proposition, *p* (*p* is false « there is a unique *x*: Sachverhalt (*x*) and *p* pictures *x* and *x* does not obtain).⁴¹

The restriction implies that Wittgenstein and Moore differ over the treatment of negation. Moore accepted the straightforward inference from negative truths to negative facts, whereas Wittgenstein was a logical atomist. For Wittgenstein, only atomic propositions, all of which are 'positive', need truth-makers to be true, and their contradictories receive a truth-value by default: if the truth-maker for an atomic *p* fails to exist, that is *eo ipso* the reason why its contradictory, 'not-*p*', is true; and if the truth-maker for *p* does exist, then 'not-*p*' will be false without further ado.⁴² But there is also an important point of agreement between Wittgenstein and Moore (pending further investigation into the metaphysics of the *Tractatus*). Wittgenstein's 'picturing' assumes the role of 'reference' and with the same effect: as long as the relation is not analysed up, it implies commitment to *mere*

³⁹ That we know the relation (are acquainted with it) is readily shown by a reflection on individual instances: we understand perfectly well, for example, what the belief is that someone is having when he believes that lions exist; and we know perfectly well what the fact is which must exist if the belief is true.

⁴⁰ The definition in the text formulates the contrast in terms of existence and being; Moore speaks of facts which have no being or are not in the Universe.

⁴¹ See Wittgenstein (1922: 4.21; 4.25). 'Sachverhalt' is naturally translated as 'state of affairs' (as in the Pears/McGuinness translation).

⁴² This formulation repeats the lucid discussion of Wittgenstein's position in Simons (2008).

possibilia—facts that do not exist or states of affairs that do not obtain. Such theories were unacceptable for Russell, because he held ‘non-existent’ to be an illegitimate category: ‘what does not exist’ must be reduced to something actual before an analysis featuring such entities can be regarded as complete. This is a well-known, standing feature of Russell’s thinking. A striking formulation, and one that fits the present context very well, is from a draft that Russell composed in 1919:

It is [...] very confusing in the case of a false proposition to say that it “means” the fact which makes it false, since we should naturally say that it means just the opposite. *Nor can we say that it means the fact which would make it true if it were true, since there is no such fact.*⁴³

As long as he believed in MRTJ in something like its original form, Russell accepted the standard account of correspondence truth, because he believed that he could specify ‘the truth-making fact’ as a function of the relevant belief-as-a-multiple-relation, thereby analysing away the apparent reference to the non-existent and rendering (C-false) ontologically innocuous. Eventually, however, he came to the conclusion that this cannot be done, a conclusion that forced him temporarily to abandon the standard account.⁴⁴ He then introduced the *duality of positive and negative facts* for the purpose of explaining truth and falsehood in a semantically innocuous way. Parallel to the distinction between positive and negative facts, there is also one between positive and negative atomic propositions. Suppose then that ‘this is red’ is a positive atomic proposition.⁴⁵ What does it specify, according to Russell’s new way of thinking? It specifies *the fact which makes the proposition true or makes it false*—I use ‘specifies’ as a neutral term between ‘means’ and ‘refers’, neither of which would be wholly appropriate in the present context. Accordingly, what we know when we understand the proposition is not some mysterious entity that the proposition supposedly refers to or pictures; what we know is what is the case if the proposition is true and what is the case if it is false. And this fact is the positive fact that this is red, if that is the fact, or the negative fact

⁴³ Russell (1919b: 8; italics added).

⁴⁴ Wittgenstein’s influence played a major role in bringing about this change in Russell’s thinking, but the details of how this happened are not relevant for the present discussion. The abandonment is temporary; as we saw above, Russell did advocate the standard account in the later 1940s.

⁴⁵ A major question for Russell is, how the distinction between positive and negative atomic propositions is drawn. There is no simple way of doing this—cf. Ayer (1952)—but possibly Russell held that allegiance to an ideal logical language would be helpful.

that this is not red, if that is a fact. And here, at least on the face of it, there is no reference to anything nonactual.⁴⁶

Having lost his belief in (C-false), Russell had to find a different account of truth and falsehood. A crucial part of the new account concerns atomic propositions. Considered abstractly, this merely says that an atomic proposition is true, only if its ‘polarity’ (its being positive or negative) matches with the ‘polarity’ of its correlated fact; and an atomic proposition is false, if there is a mismatch of ‘polarities’.⁴⁷

Russell’s new theory of truth raises hard questions about the semantic and ontological aspects of ‘polarity’. How is the correlation of atomic propositions (judgements) and their true- and false-making facts effected? What is the structure of positive and negative atomic facts? And, in general, what is the notion of fact that figures in this account of truth?

7 The idea of bipolarity

The idea of polarity comes from Wittgenstein. He had introduced what he called ‘bipolarity’ as the distinctive characteristic of propositions in his *Notes on Logic* of 1913, a work that Russell studied in detail and many of whose ideas found their way into Russell’s lectures on logical atomism in 1918. The most abstract formulation of bipolarity is this:

(BIPOLARITY) Every proposition is essentially true-false.⁴⁸

Wittgenstein uses bipolarity to undermine what he sees as a mistaken conception of propositions as names of complexes or situations, a conception that he claims to find in Frege and Russell. The point is this. Suppose one distinguishes names from propositions on the grounds that a name is correlated with a *thing* (individual), whereas a proposition is correlated with a *situation*, which is complex. This, however, would not be sufficiently distinctive of propositions, unless one immediately added that the situation *may not in fact be there*, because the proposition may be false. And this may be taken to suggest—and was so taken by Wittgenstein and later by Russell—that

⁴⁶ Note that the two logical atomists, Russell and Wittgenstein, agree on an important structural point about ‘truth-making’. They both think that the explanation of why an atomic *p* is true (false) is exactly the same explanation of why its contradictory is false (true). On this point, they disagree with Moore. But they disagree with each other on what the explanation is.

⁴⁷ Since Russell did not believe that the truth of all logically complex propositions could be traced back to atomic propositions, further clauses are needed to deal with certain non-atomic cases, like general and existential propositions; see Russell (1918a: 207–208), where the matter is discussed from the point of view of ‘truth-making’ rather than truth-definition.

⁴⁸ Wittgenstein (1913: 94).

what really distinguishes propositions from names is that *propositions are true or false of reality*; this is another formulation for bipolarity.

According to Wittgenstein's *Notes*, the sense of a proposition consists in its having these two poles, corresponding to its truth and its falsehood, respectively. In 1913, however, Wittgenstein was not yet in a position to explain what this sense consists in. A year later he had found the solution:

We can say straight away: Instead of: this proposition has such and such a sense: this proposition represents such and such a situation. (2.10.1914; Wittgenstein 1961: 8e)

Rather than renouncing bipolarity, this is meant to reveal its underlying mechanism: picturing is logical portrayal (*ibid.*) and

[o]nly in this way can the proposition be true or false: It can only agree or disagree with reality by being *a picture* of a situation (*ibid.*)

In the *Tractatus*, picturing is restricted to elementary propositions. This yields the account of truth and falsehood that we met in the previous section. Inspired by Wittgenstein's pre-war *Notes*, Russell explained in 1918:

It is very important to realize such things, for instance, as that *propositions are not names of facts*. It is quite obvious as soon as it is pointed out to you, but as a matter of fact I never had realized it until it was pointed out to me by a former pupil of mine, Wittgenstein. It is perfectly evident as soon as you think of it, that a proposition is not a name for a fact, from the mere circumstance that there are *two* propositions corresponding to each fact. (1918a: 167; emphasis added)⁴⁹

⁴⁹ That is, every atomic fact has a positive proposition, *p*, and a negative proposition, *not-p*, corresponding to it. Russell's claim about self-evidence is quite misleading. It may be obvious that a proposition is not a name from the fact that there are two propositions correlated with every fact; what is not obvious, though, is that there actually *are* two propositions correlated with every fact. Moore, example, would not have found that obvious at all. What he found obvious was, first, that every proposition (that is, every 'what is believed') is associated with just one 'fact'; and second, that it is the existence vs. non-existence of this associated fact that decides whether the belief is true or false.

For the reason explained above, however, Russell could not follow Wittgenstein's explication of bipolarity in terms of picturing. Instead, what informs Russell's 1918-version of logical atomism on this point is the following idea:

A proposition points towards a fact (if it is true) or away from it (if it is false).⁵⁰

Unfortunately, when Russell gave his lectures, he was not yet in a position to explain what this pointing towards or pointing away consisted in. In the lectures, he continued to endorse the basic intuition behind MRTJ, according to which a judging mind, when it believes something, is multiply related to a number of things. So, minimally, the Russell of 1918 is committed to this:

A judgement can be true or false of reality only by the judging mind's being related to the constituents of the fact which makes the judgment true or false.⁵¹

Recall that MRTJ sought to explain judgement in a way that introduces nothing over and above a judging subject and a multiplicity of objects. But by time he gave the lectures on logical atomism, Russell had renounced older versions of MRTJ as 'a little unduly simple' (1918a: 199). The minimal condition on judgement was therefore no longer adequate.

8 The psychological theory of judgement

Russell's subsequent reflections on the semantics of (bi)polarity break away from the framework of MRTJ and move in a completely new, *psychological*, direction; accordingly, I will refer to this development as a psychological theory of judgement (PTJ).

One important factor here is Russell's long-time confrontation with *neutral monism*. This eventually convinced Russell that the judging mind as what he called a 'pin-point Subject' is at best a logical construction and not an entity that could be given.⁵² This undermines MRTJ, because, as Russell himself points out, if no such pin-point subject is available, there will be nothing to bring a number of worldly objects together so as to form a judgement involving those objects.⁵³ This

⁵⁰ See Russell (1918a: 185); and slightly later: 'The essence of the proposition considered as a symbol is its duality of possible relation to fact' (1918b: 268).

⁵¹ See Russell (1919a: 295).

⁵² For the term 'pin-point Subject', see (Russell 1918b: 265, 268); Russell (1959: Chapter XII) is a useful retrospective reflection on neutral monism and its influence. For a critical discussion Russell's version of neutral monism, see Banks (2014: Chapter 4).

⁵³ See Russell (1919a: 295).

conclusion now forces upon Russell a *representational* account of experience, which holds, among other things, that judgement or belief involves *contents* that reside ‘in the mind’ (1919a; 1921: Lecture XIII).

PTJ professes to be a psychologically realistic theory, but it has a ‘logical’ or ‘formal’ side to it as well. Here Russell’s thinking continues to be influenced by bipolarity and Wittgenstein. A judgement or belief occurs, not when a subject judges or believes something (whatever the mechanism) but when there occurs an *image-proposition* that is accompanied by an appropriate *feeling*. There are also *word-propositions*, but these are secondary and derivative from image-propositions.

The semantic story now comes to focus on the image-proposition. A proposition, Russell holds, is a *fact* and ‘[t]his is why it can express a fact’ (1918b: 268). The idea is purebred Wittgenstein, of course, but Russell’s first attempts to work it out continue to refer to positive and negative facts, as in the following:

Return to [the proposition] “Smith hates Jones”. There are two possible relations of this to the fact which determines truth and falsehood: Smith may have or not have that relation to Jones which is the meaning of the word “*hates*”. This is the formal basis of the possibility of using sentences to symbolize facts: the sentences *are* facts, whose form is the same as or opposite to that of the facts they are to symbolize. Hence truth and falsehood. (1918b: 270–271)

Russell soon came to see, however, that this will not do. The reason is that there is a fundamental lack of parallelism between image-propositions, on the one hand, and the facts that are supposed to be their truth-makers and false-makers. An image-proposition, Russell notes, is always a positive fact: ‘There is no visualizing “A-not-to-the-left-of-B”. When we attempt it, we find ourselves visualizing “A-to-the-right-of-B” or something of the sort’ (Russell 1919a: 304).⁵⁴ This applies to word-propositions as well; even a proposition like ‘Smith does not hate Jones’, Russell argues, is itself a positive fact. He does claim that there is a difference between word-propositions that are verified by positive facts and word-propositions that are verified by negative facts.⁵⁵ Nevertheless, the point remains that no purely ‘formal’ explanation is forthcoming of the duality of truth and falsehood, for the simple reason that image- and word-propositions are always positive facts, whereas their truth- and false-makers are supposed to be either positive or negative.

⁵⁴ See also Russell (1921: 275–277).

⁵⁵ See Russell (1919a: 304; 1921: 276–277).

For Wittgenstein, that propositions are facts gives the essence of the idea of proposition as picture: a proposition is a (logical) picture of a situation because they share a common structure, that of a fact.⁵⁶ In Russell's semantics, by contrast, the notion that propositions are facts and pictures plays only a limited role, as the idea of isomorphism as the basis of representation applies only in the simplest case where there is a resemblance between an image-proposition and its truth-making fact (1921: 274); beyond that, there will be no formal similarity or isomorphism on which to build the difference between truth and falsehood.⁵⁷ When Russell explains the family of ideas that we have been exploring here, he is careful to attribute the basic idea to Wittgenstein.⁵⁸ But to my knowledge, he nowhere indicates that there actually are deep differences between his own use of bipolarity and Wittgenstein's. More importantly, Russell's explanations leave us in the dark about two important questions: What, exactly, is the semantics of bipolarity on Russell's version of the notion? Can that semantics be given an actualist interpretation in a way that accords with Russell's intentions?

In a way, PTJ signifies a return to quality dualism. In that theory, negation is not a semantic phenomenon; at least, it cannot be given a 'formal' explanation. Russell does not quite draw this conclusion, but he acknowledges that image-propositions 'do not allow any duality of content' (1919a: 304). This applies to word-propositions as well. What there is, though, are *belief* and *disbelief*—although now that no role is given to the pin-point subject, it remains to be spelled out what, exactly, belief is and how it differs from disbelief, on the one hand, and from a plethora of other 'attitudes', on the other.⁵⁹

9 Further developments in Russell's psychological theory

This issue of quality dualism gives an indication of a new direction in Russell's thinking. The notion of meaning, he now thought, is primarily a psychological one, and should be explicated in terms of causes and effects. Given this, a 'logical' or 'formal' theory of meaning of the sort that he had envisaged earlier becomes less and less prominent for Russell. One victim of this development is the notion of fact, but the changing role of 'fact' in Russell's thinking can also be approached from the formal side. What follows is a quick sketch of what is involved in that formal side.

⁵⁶ Wittgenstein (1922: 2.14; 2.18; 3.14; 3.142).

⁵⁷ Cf. Russell's retrospective comments on Wittgenstein's influence in Russell (1959: 113–114).

⁵⁸ See Russell (1918a: 167; 1921: 271–273).

⁵⁹ See Russell (1921: Lecture XII) for a psychologically 'realistic' discussion of what is meant by belief.

In 1927, Russell published a book, *An Outline of Philosophy*, which is a sort of update on the much more famous *The Problems of Philosophy*. He now distinguishes four aspects in ‘the subject of truth and falsehood’ (1927: 208). One of these is a formal theory, whose question is this: ‘Given the meaning of the component words, what decides whether a sentence is true or false?’ (ibid.) This question is familiar to all analytical philosophers. Russell replies as follows:

E. g. ‘Brutus killed Caesar’ is true because of a certain fact; what fact? The fact that Brutus killed Caesar. This keeps us in the verbal realm, and does not get us outside it to some realm of non-verbal fact by which verbal statements are verified. (ibid.)

Russell’s point here might be more properly expressed by saying that, according to him, facts are the ‘tautological accusatives’ of true sentences.⁶⁰ Whatever the proper formulation, however, the point is that in advancing the above claim, Russell has drifted quite far from the MRTJ-conception of facts as complex objects and the concomitant explanation of truth as consisting in correspondence with such facts *qua* objects. The question arises: What has brought about this conceptual change?

There are at least two explanations that could be cited here. Both are plausible and not exclusive of each other. The first one refers to the formal development that we envisaged in the previous section. Plausibly, given Russell’s premises, there is no other way of drawing the distinction between positive and negative facts than via their roles as truth- and false-makers for certain kinds of sentences:

‘Brutus killed Caesar’ is true if and only if it is a fact that Brutus killed Caesar.

‘Brutus didn’t kill Caesar’ is true if and only if it is a fact that Brutus didn’t kill Caesar.

The hypothesis here is simply that reflection on the formal distinction between positive and negative facts led Russell to a ‘deflationary’ conception of facts, which consists in the provision of such biconditionals. Some evidence for the hypothesis is provided by Russell’s discussion of the formal theory of truth and falsehood in *The Analysis of Mind*, which ends with the following remark:

⁶⁰ Cf. here Armstrong (1997: 19).

I do not believe that the above formal theory is untrue, but I do believe that it is inadequate. It does not, for example, throw any light upon our preference for true beliefs rather than false ones. This preference is only explicable by taking account of the causal efficacy of beliefs, and of the greater appropriateness of the responses resulting from true beliefs. (Russell 1921: 278)

The general line of thought that I am here tentatively attributing to Russell is more familiar from Ramsey's 'redundancy theory of truth' (Ramsey 1927). In making his famous dictum about there being no separate problem of truth over and above the analysis of judgement, Ramsey was in fact downplaying the importance of a 'formal theory of truth' and emphasizing, if only indirectly, the importance of a 'causal theory': once we get clear about what is involved in the judgement that aRb (this being the task that we accomplish in analysing the judgement), we can thereby see what is involved in a formal theory of truth: we can see, for example, that if a stands in R to b , then the judgement that a has R to b is true; and if a does not stand in R to b , then the judgement is false. We can add, if we like, that the judgement is true if there exists a corresponding fact that a has R to b ; but this is just a paraphrase of ' a has R to b ' (Ramsey 1927: 158–159). This is the deflationary conception of facts. What the analysis of judgement reveals, on the other hand, is minimally this. When I judge that so-and-so, this involves both a subjective and an objective factor: my present mental state or words or images in my mind, on the one hand, and a fact that so-and-so, or a proposition that so-and-so, on the other hand (Ramsey 1927: 153). There is also the relationship between these two factors, and this has a formal and a 'material' side to it. The formal side, Ramsey held, following Wittgenstein, is essentially intractable. This is because we cannot know anything a priori about the logical forms of atomic propositions, save that they exhibit the general form of a proposition ('such and such is the case').⁶¹ This is just a different way of expressing the above conclusion regarding the formal theory of truth. There then remains the material side of the relationship. This is where the task of the analysis of judgement lies, and such an analysis can be given along broadly 'pragmatist' lines that uses belief and action to throw light on truth vs. falsehood. Though this is pragmatism of sorts, the framework is thoroughly Russellian, an influence about which Ramsey himself is explicit.⁶²

⁶¹ See Ramsey (1925).

⁶² 'In conclusion, I must emphasise my indebtedness to Mr. Wittgenstein, from whom my view is derived. Everything that I have said is due to him, except the parts which have a pragmatist tendency. [...] My pragmatism is derived from Mr. Russell; and is, of course, very vague and undeveloped' (Ramsey 1927: 170). Wittgenstein's rejection of 'pragmatism' (he continues to call it

The second explanation focuses on the semantics or logic of ‘fact’. That sentences must be distinguished from names was one of the points that Russell got from Wittgenstein (cf. Sect. 7 above). Wittgenstein would not have liked the formulation, but Russell thought of the distinction as one of type and saw it as reflecting a distinction between kinds of entities. Accordingly, since sentences purport to state facts (or ‘call up beliefs’, as Russell puts it in the following quotation), the peculiarity of sentences lies in the nature of facts and in their having constituents of different kinds. In particular, it lies in the *difference between relations (including attributes) and their terms*:

Observe that “hates” has meaning in quite a different way from “Smith” and “Jones”. “Smith” and “Jones” are to call up *images*. But “hates”, by itself, is to call up nothing. When it occurs with other words, so as to create a *fact*, its business is to call up a *belief* (whatever that may be). The word “hates” by itself is strictly devoid of meaning. The purpose of the word “hates” is merely to produce a certain relation between “Smith” and “Jones”, so that there may be a fact about “Smith” and “Jones” to correspond with the fact about Smith and Jones. (Russell 1919b: 271)

To mean a term of a relation is to name it, whereas to mean a relation is to assert something that may or may not be a fact. Accordingly, relations—and with them, facts—lose the substantiality of their terms (objects, ‘simples’):

If what we take to be simple is really complex, we may get into trouble by naming it, when what we ought to do is to assert it. For example, if Plato loves Socrates, there is not an entity “Plato’s love for Socrates”, but only the fact that Plato loves Socrates. And in speaking of this as “a fact”, we are already making it more substantial and more of a unity than we have any right to do. (Russell 1924: 337)

In this way, facts cease to be entities in the sense of MRTJ, and they can no longer do the theoretical work that they did in the correspondence theory of truth. One can continue to assert that truth is a matter of correspondence, but if one does so, what one thereby asserts is a theory that assumes a ‘deflationary’ account of facts.

psychology, of course) is in evidence in his comments on Russell’s queries about the *Tractatus*. See Wittgenstein (1995: 125).

In his later works, Russell often applies a somewhat different reconciliation of truth and facts. He continues to explain truth in terms of correspondence and facts,⁶³ but now he construes facts as *events*, as in the following quotation from *An Inquiry into Meaning and Truth*, where he discusses the idea of facts' making propositions true:

First, as to 'facts': they are not to be conceived as 'that grass is green' or 'that all men are mortal'; they are to be conceived as occurrences. We shall say that all percepts are facts, but according to the realist view they are only some among facts. [...] Percepts, in this view, may be defined as events having a certain kind of spatio-temporal relation to a living body with suitable organs. (1940: 284–285)

Thus, in Russell's conception of facts, the following development takes place:

- (1) Facts are construed as complex objects (MRTJ).
- (2) Facts are construed as 'tautological accusatives' of true statements, which leads to a purely linguistic notion of fact (1920s)
- (3) Facts are construed as or replaced by events (1940s)

This development is also significant for the topic of negative judgement. For if truth-making facts are really events, then it is no longer plausible to try to locate negation in the world. Negation thus becomes something psychological. And this is exactly what we find Russell arguing for.⁶⁴ Negative judgement only arises in connection with experience ('perception') and 'perception only gives rise to negative judgment when the correlative positive judgment had already been made or considered. When you look for something lost, you say "no, it's not there"; after a flash of lightning you may say "I have not heard the thunder"' (1948: 138). Of course, in addition to such judgements, there is also something 'in the way of objective fact' (*ibid.*) that grounds the truth of the negative judgement. It remains unclear, however, whether this explanatory strategy suffices to dispense with negative facts in some sense of that term (negativity or negation in an ontological sense). I may truly say, 'this is not red', because this is, say, blue. But as Russell himself had observed some three decades earlier, this is only because what is blue is also not red. Russell now picks up the subject afresh, but despite the agility of his mind, the discussion remains inconclusive at best.

⁶³ See Russell (1927: 210; 1940: Chapters XVI and XVII; 1948: Chapters VIII and IX).

⁶⁴ See Russell (1940: Chapter V; 1948: Chapter IX).

10 Conclusion

A broadly Fregean conception of negative judgement as consisting of assertion plus negative proposition is often regarded as the orthodox conception of negative judgement within analytic philosophy. Our discussion of Russell's theories of negative judgement has revealed, if not an alternative to Frege's view, then at least a rather more varied landscape even within analytic philosophy.

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